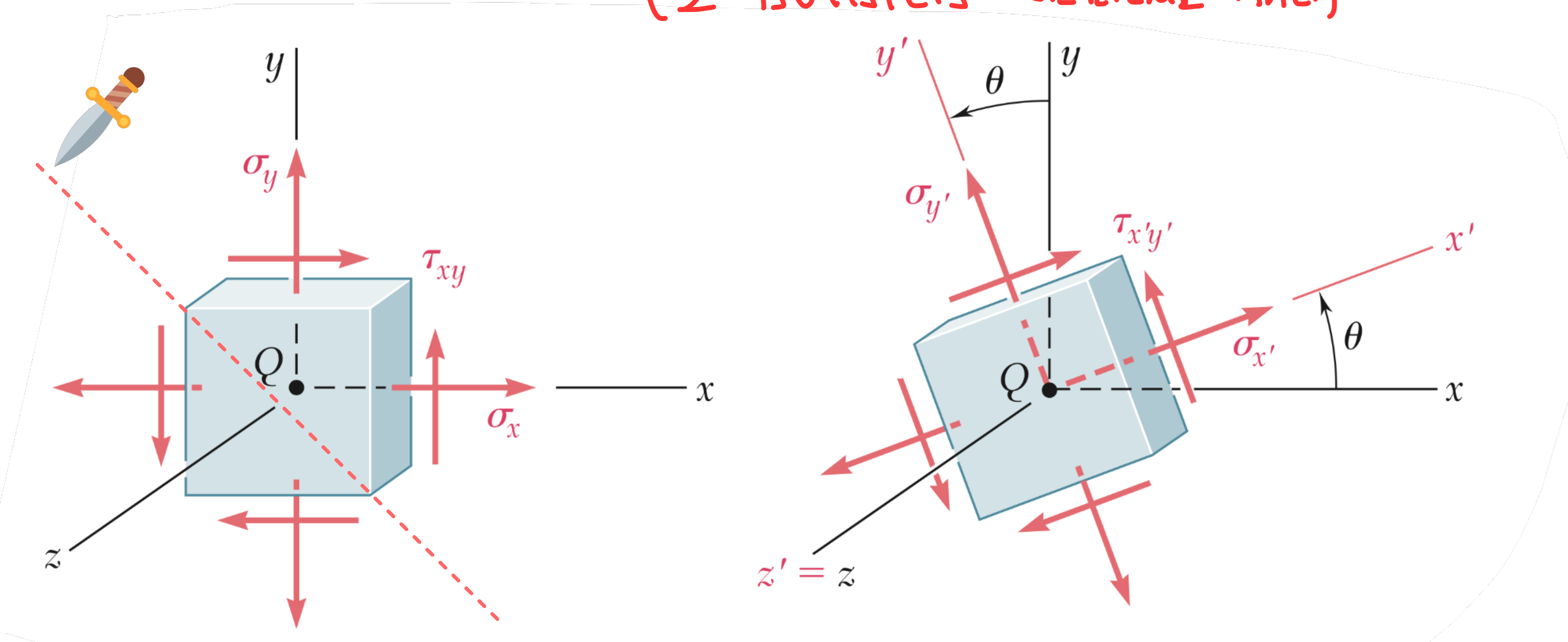
GERİLME – GERİNME DÖNÜŞÜMLERİ

(2 BOYUTLU GERİLME HALİ)



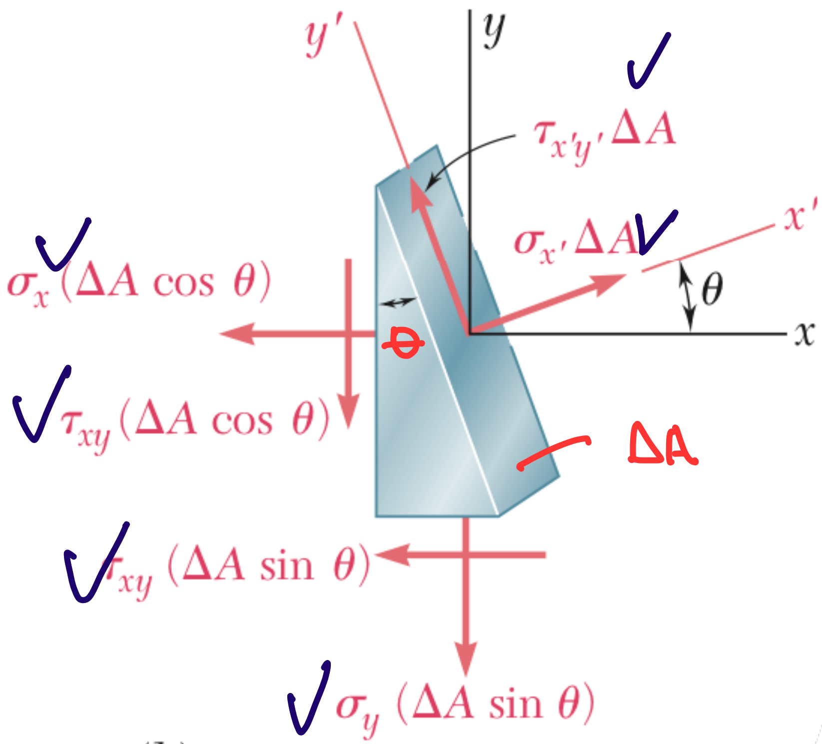
$\tau_{x \; y}$

x → YÜZEYİN NORMALİ

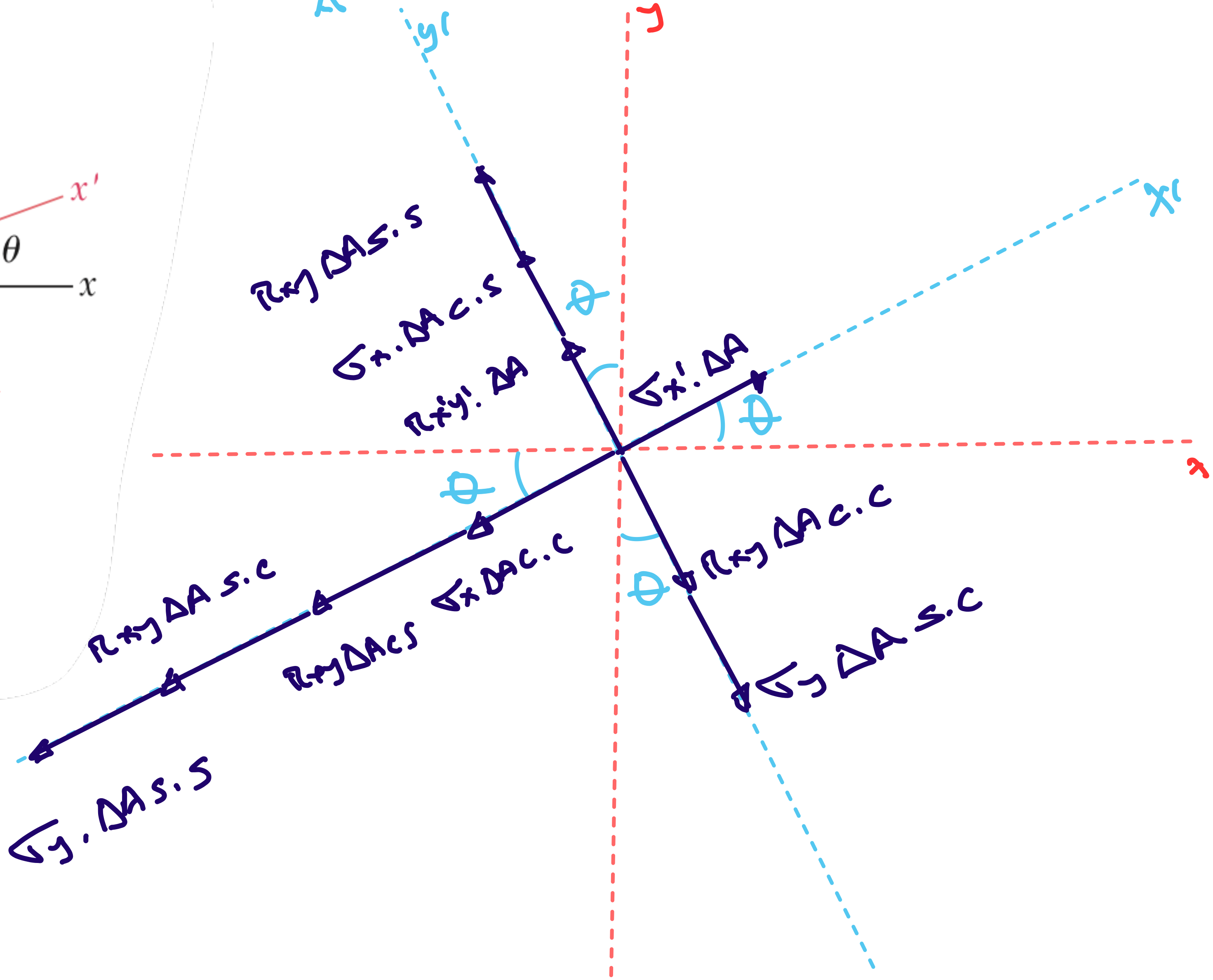
y → DOĞRULTU

AMAÇ $\left.\begin{matrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \end{matrix}\right\}$ $\begin{matrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{matrix}$ cinsinden yazmak

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma . A$$



(b)



$$\sum F_{x'} = 0: \quad \sigma_{x'}\,\Delta A - \sigma_x(\Delta A \cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta$$
$$-\sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\,\Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta$$
$$-\sigma_y(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta = 0$$

$$\sigma_{x'} = \sigma_x . \cos^2\theta + \sigma_y . \sin^2\theta + 2 . \tau_{xy} \cos\theta \sin\theta$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\cos\theta . \sin\theta + \underbrace{\tau_{xy}\cos^2\theta - \tau_{xy}\sin^2\theta}_{\tau_{xy}(\cos^2\theta - \sin^2\theta)}$$

$$\boxed{\begin{array}{ll} \sin 2\theta = 2 . \cos\theta \sin\theta & \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \sin^2\theta = \dfrac{1 - \cos 2\theta}{2} & \cos^2\theta = \dfrac{1 + \cos 2\theta}{2} \end{array}}$$

$$\sigma_{x'} = \sigma_x \cdot \cos^2\theta + \sigma_y \cdot \sin^2\theta + 2 \cdot \tau_{xy} \cos\theta \sin\theta$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\cos\theta \cdot \sin\theta + \underbrace{\tau_{xy}\cos^2\theta - \tau_{xy}\sin^2\theta}_{\tau_{xy}(\cos^2\theta - \sin^2\theta)}$$

$$\sigma_{x'} = \frac{\sigma_x}{2} + \frac{\sigma_x \cdot \cos 2\theta}{2} + \frac{\sigma_y}{2} - \frac{\sigma_y \cdot \cos 2\theta}{2} + \tau_{xy} \cdot \sin 2\theta$$

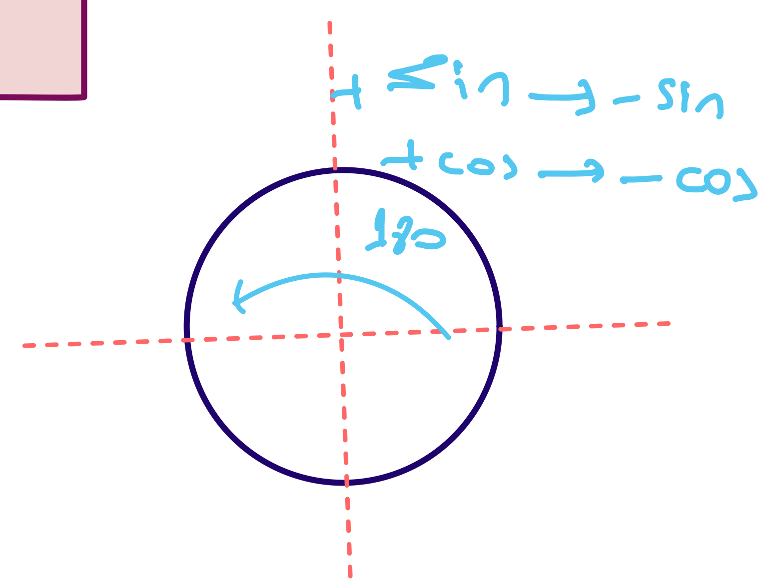
$$\boxed{\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \cdot \sin 2\theta}$$ — A

$$\boxed{\tau_{x'y'} = -\frac{(\sigma_x - \sigma_y)}{2} \cdot \sin 2\theta + \tau_{xy} \cdot \cos 2\theta}$$ — B

$\sigma_{y'}$ için $\theta = \theta + 90$ yazılabilir.

Bizde $\theta = 2\theta \longrightarrow \sigma_{y'}$ için $\theta = 2\theta + 180$

$\cos(2\theta + 180)$ $\sin(2\theta + 180)$



$$\boxed{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2} \cdot \cos 2\theta - \tau_{xy} \cdot \sin 2\theta}$$

C

A + C DENKLEMİ $\equiv$ $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$

GERİLME TOPLAMLARI YÖNELİMDEN BAĞIMSIZDIR

ASAL GERİLMELER

$$\Bigg(\sigma_{x'} - \underbrace{\frac{\sigma_x + \sigma_y}{2}}_{\sigma_{ort}}\Bigg)^2 = \left(\frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \sin 2\theta\right)^2$$

$$\left(\tau_{xy'}\right)^2 = \left(-\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta\right)^2$$

$\rightarrow \theta$ yok edilir

$$\Bigg(\sigma_{x'} - \underbrace{\frac{\sigma_x + \sigma_y}{2}}_{\sigma_{ort}}\Bigg)^2 + \tau_{xy'}^{2} = \underbrace{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

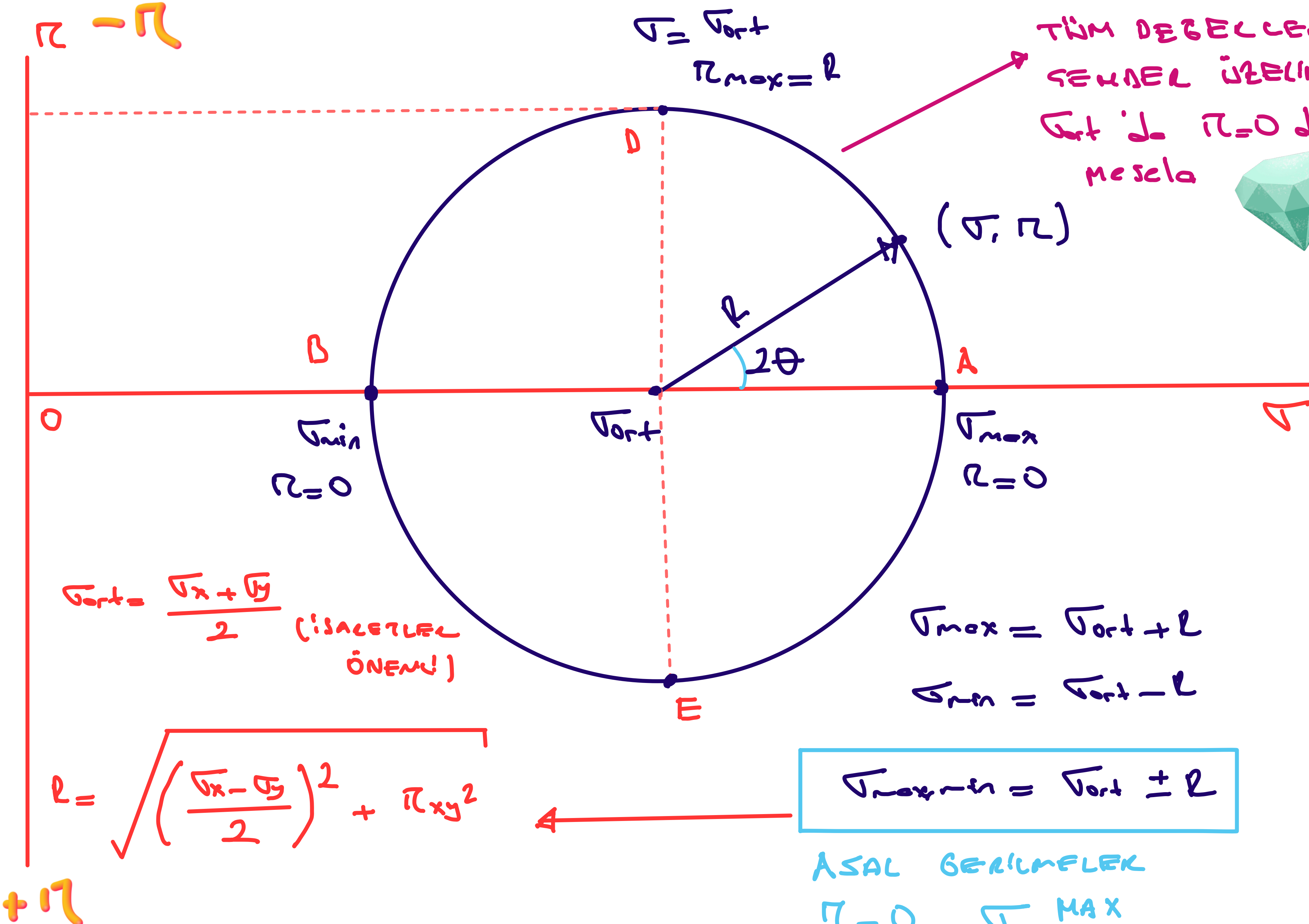
$$A^2 + B^2 = C^2 \quad R^2$$

$x^2 + y^2 = R^2 \longrightarrow y = \sqrt{R^2 - x^2}$

$$\left(\sigma_{x'} - \sigma_{ort}\right)^2 + \tau_{x'y'}^{\,2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\sigma_{ort} = \frac{\sigma_x + \sigma_y}{2}$$

UNUTMA;
TÜM DEĞERLER ÇEMBER ÜZERİNDE
σ_{ort}'da $\tau=0$ değil mesela



$$\sigma_{ort} = \frac{\sigma_x + \sigma_y}{2}$$ (İŞARETLER ÖNEMLİ!)

$$\sigma_{max} = \sigma_{ort} + R$$

$$\sigma_{min} = \sigma_{ort} - R$$

$$\boxed{\sigma_{max,min} = \sigma_{ort} \pm R}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

ASAL GERİLMELER

$\tau = 0$ $\quad \sigma$ MAX MIN

ASAL DÜZLEM İÇİN $\tau = 0$

$$\tau_{x'y'} = -\frac{(\sigma_x - \sigma_y)}{2} \cdot \sin 2\theta + \tau_{xy} \cdot \cos 2\theta = 0$$

$$\Rightarrow \boxed{\tan 2\theta_p = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}}$$

(Principal)

KAYMA DÜZLEMİ İÇİN (τ_{max}) $\sigma = \sigma_{ort}$

$$\sigma_{x'} = \underbrace{\frac{\sigma_x + \sigma_y}{2}}_{\sigma_{ort}} + \underbrace{\frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \cdot \sin 2\theta}_{=0 \text{ olmalı}}$$

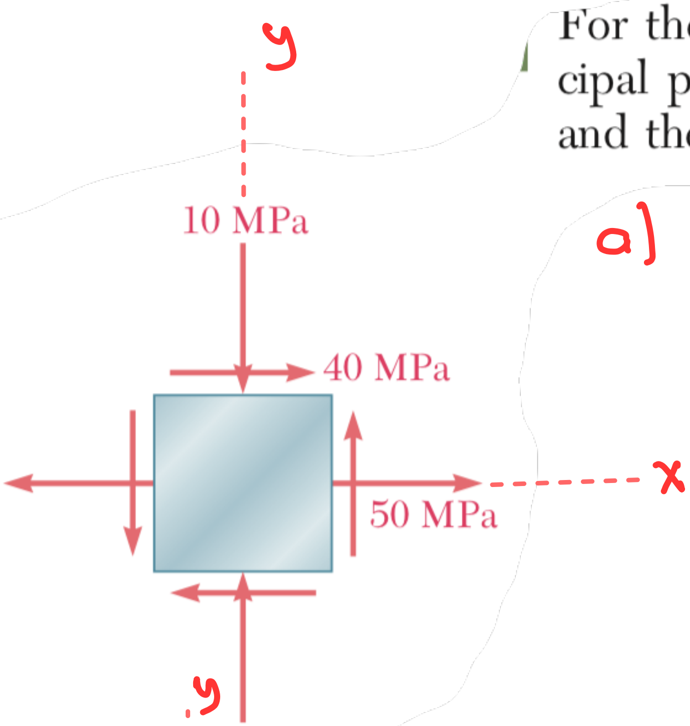
$$\Rightarrow \boxed{\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2 \cdot \tau_{xy}}}$$

(Shear)

BİR AÇI, DİĞER AÇININ TERSİ VE NEGATİFİ İSE, O AÇI İLE DİĞER AÇI ARASINDA 90° FARK VARDIR.

$\longrightarrow \quad 2\theta_s = 90 + 2\theta_p \quad \longrightarrow \quad \boxed{\theta_s = 45 + \theta_p}$

For the state of plane stress shown in Fig. 7.11, determine (*a*) the principal planes, (*b*) the principal stresses, (*c*) the maximum shearing stress and the corresponding normal stress.



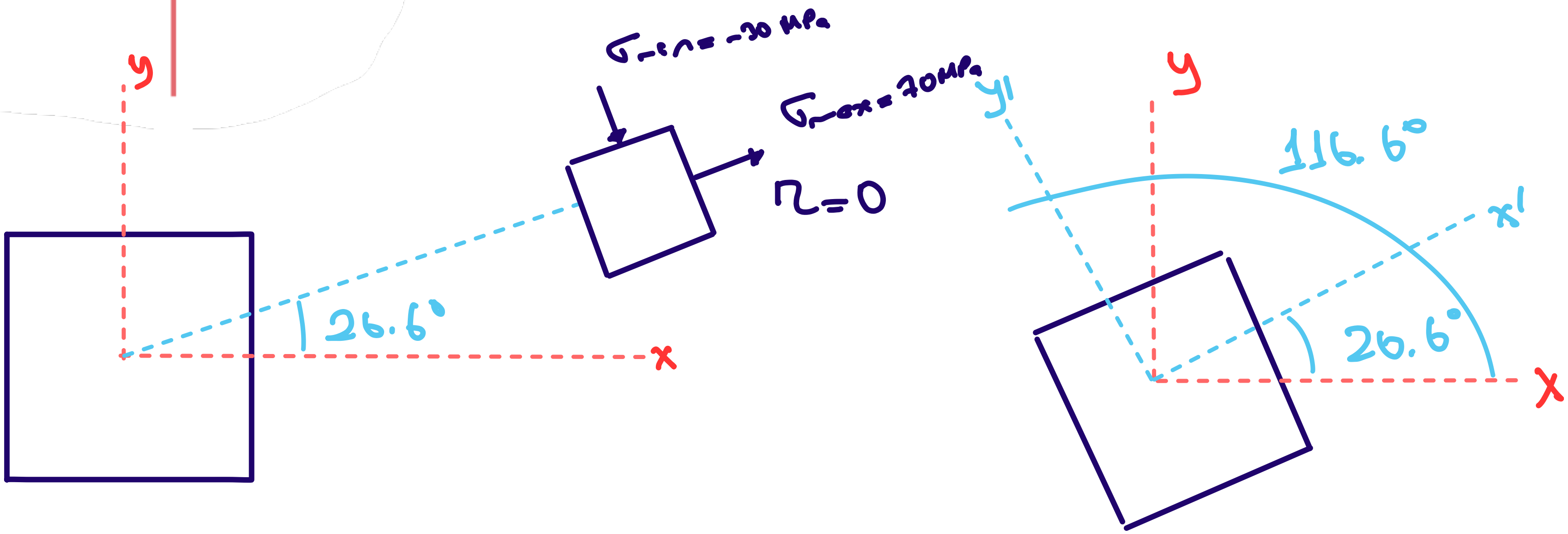
a) $\tan 2\theta_p = \dfrac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$

$\sigma_x = +50 \text{ MPa}$

$\sigma_y = -10 \text{ MPa}$

$\tau_{xy} = +40 \text{ MPa}$

$\Rightarrow \quad \theta_p = 26.60^\circ$



b.) $\sigma_{max,min} = \sigma_{ort} \pm R = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= +70\ MPa$

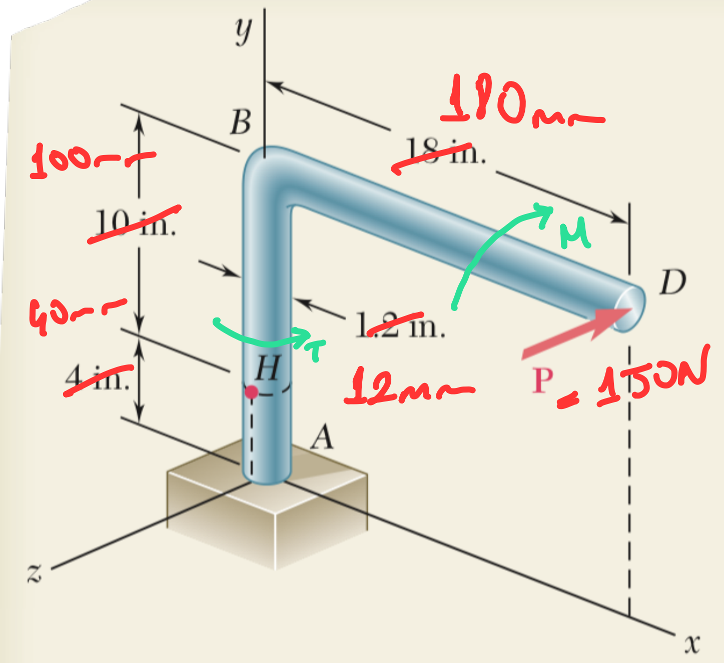
$\ \ \ -30\ MPa$

c) $\tau_{max} = R = 50\ MPa$

$\theta_p = 26.6°$ → KAYMA DÜZLEMİ için

$\theta_s = 45° \pm \theta_p \Rightarrow \theta_s = -18.4°$

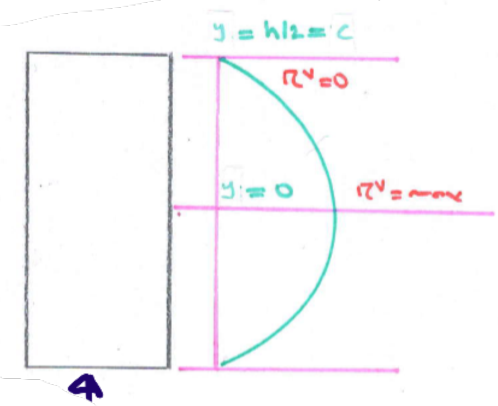
MOHR ÇEMBERİ



SAMPLE PROBLEM 7.1

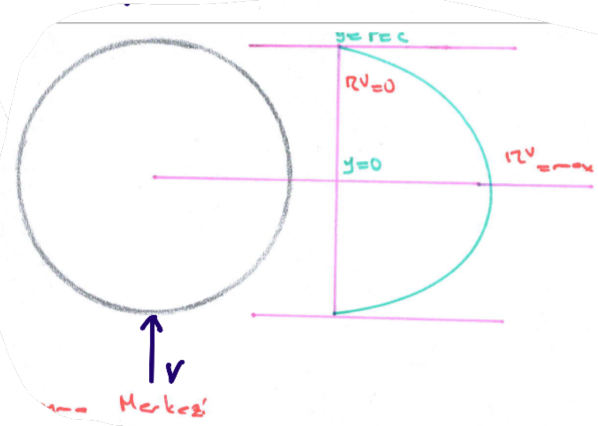
A single horizontal force **P** of magnitude 150 lb is applied to end D of lever ABD. Knowing that portion AB of the lever has a diameter of 1.2 in., determine (*a*) the normal and shearing stresses on an element located at point H and having sides parallel to the x and y axes, (*b*) the principal planes and the principal stresses at point H.

	M	T	V	N
P@H	✓ σ^M	✓ τ^T	x	x

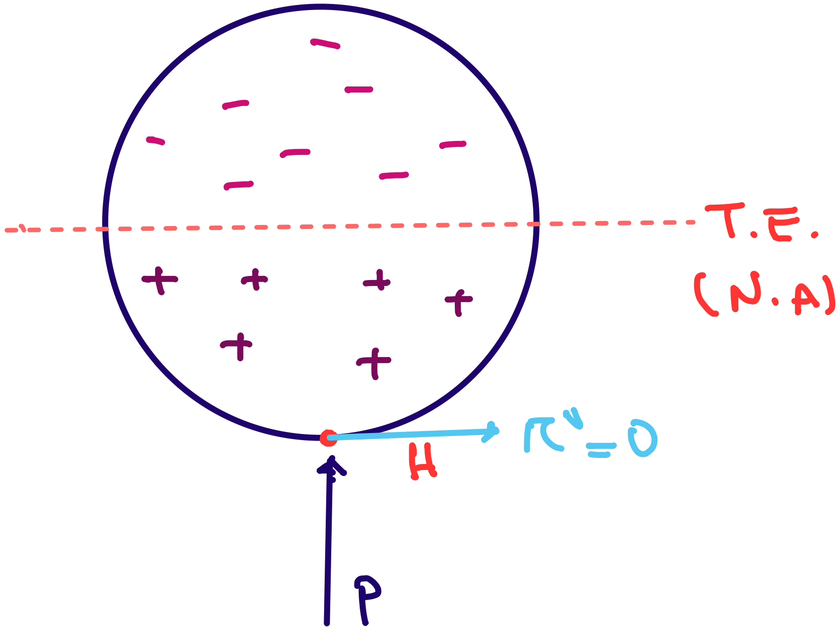


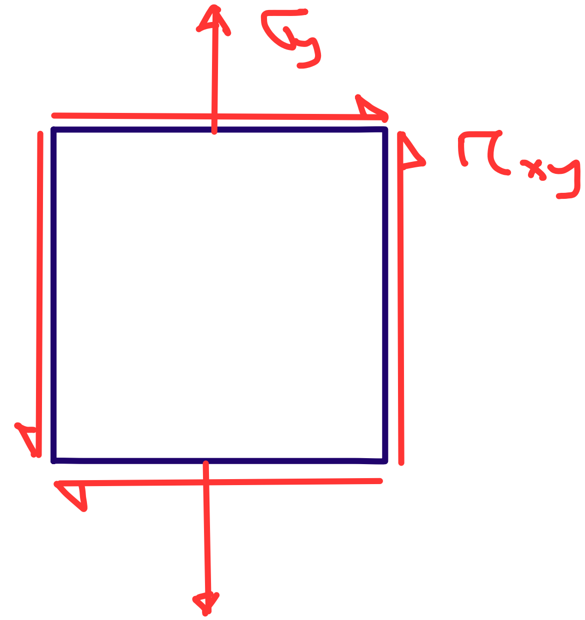
4

$$\tau^V = \frac{3V}{2A}\left[1 - \frac{y^2}{c^2}\right]$$



$$\tau^V = \frac{4V}{3A}\left[1 - \frac{y^2}{c^2}\right]$$





$$\sigma^M = \frac{M.c}{I} = \frac{M.D}{2I}$$

$$\tau^T = \frac{T.c}{J} = \frac{T.D}{2J}$$

$$M = P.100\,mm = 150 \times 100 = 15000\ Nmm$$

$$T = P.180\,mm = 150 \times 180 = 27000\ Nmm$$

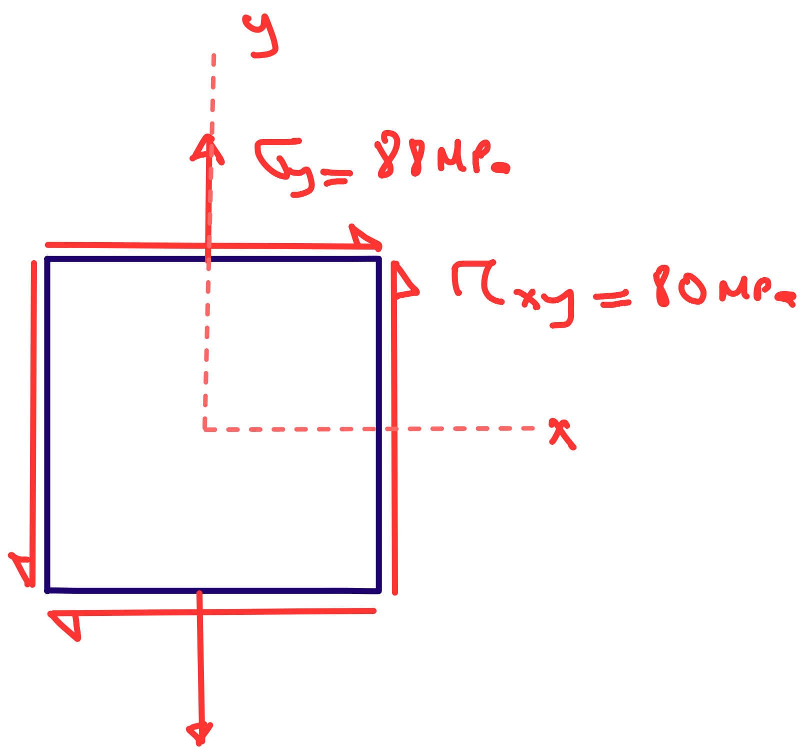
$$I = \frac{\pi D^4}{64} = \frac{\pi . 12^4}{64} = 1017.87\ mm^4$$

$$J = \frac{\pi D^4}{32} = 2 \times I = 2035.74\ mm^4$$

$$\sigma^M = \frac{15 \cdot 10^3 \cdot 12}{2 \cdot 1017.87} = 88.42\ MPa = 88\ MPa$$

$$\tau^T = \frac{27 \cdot 10^3 \times 12}{2 \cdot 2035.74} = 79.57\ MPa = 80\ MPa$$

1.) ELDEKİLER YAZILIR



$\sigma_x = 0$

$\sigma_y = +88\ MPa$

$\tau_{xy} = +80\ MPa$

2.) KURALA GÖRE X ve Y KOORDİNATLARI YAZILIR

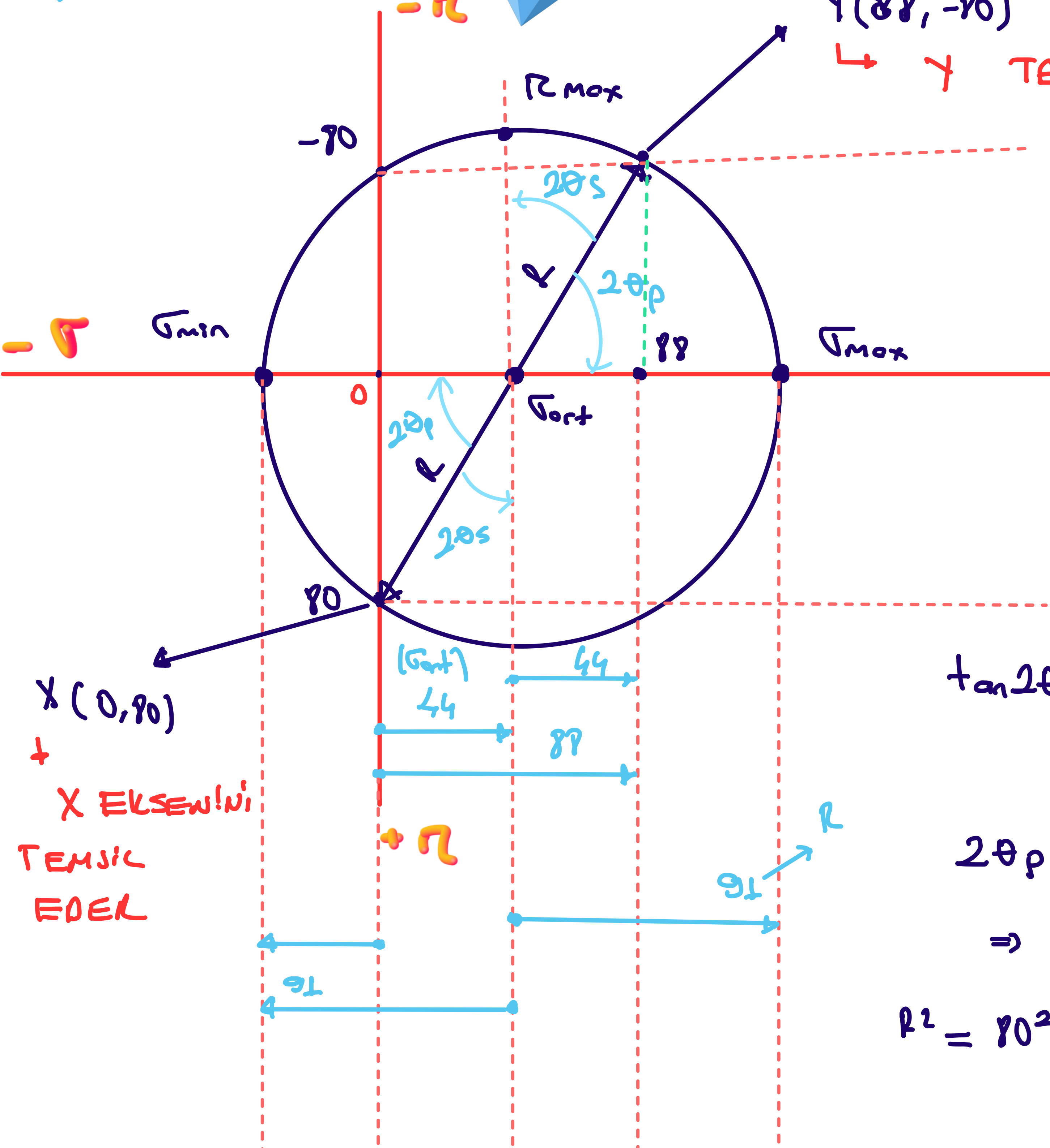
$X(\sigma_x, (+)\tau_{xy})$ $\qquad$ $Y(\sigma_y, \underline{(-)\tau_{xy}})$

τ_{xy} Y'de işaret değişir.

3.) ÇEMBER ÇİZİLİR

X(0, 80)

Y(88, −80)



$$\sigma_{ort} = \frac{\sigma_x + \sigma_y}{2} = \frac{0+88}{2} = 44 \text{ MPa}$$

İŞARETLER ÖNEMLİ

$$\tan 2\theta_p = \frac{80}{44} \Rightarrow 2\theta_p = 60^\circ$$

$$\Rightarrow \theta_p = 30^\circ$$

$$2\theta_p + 2\theta_s = 90^\circ$$

$$\Rightarrow \theta_s = 15^\circ$$

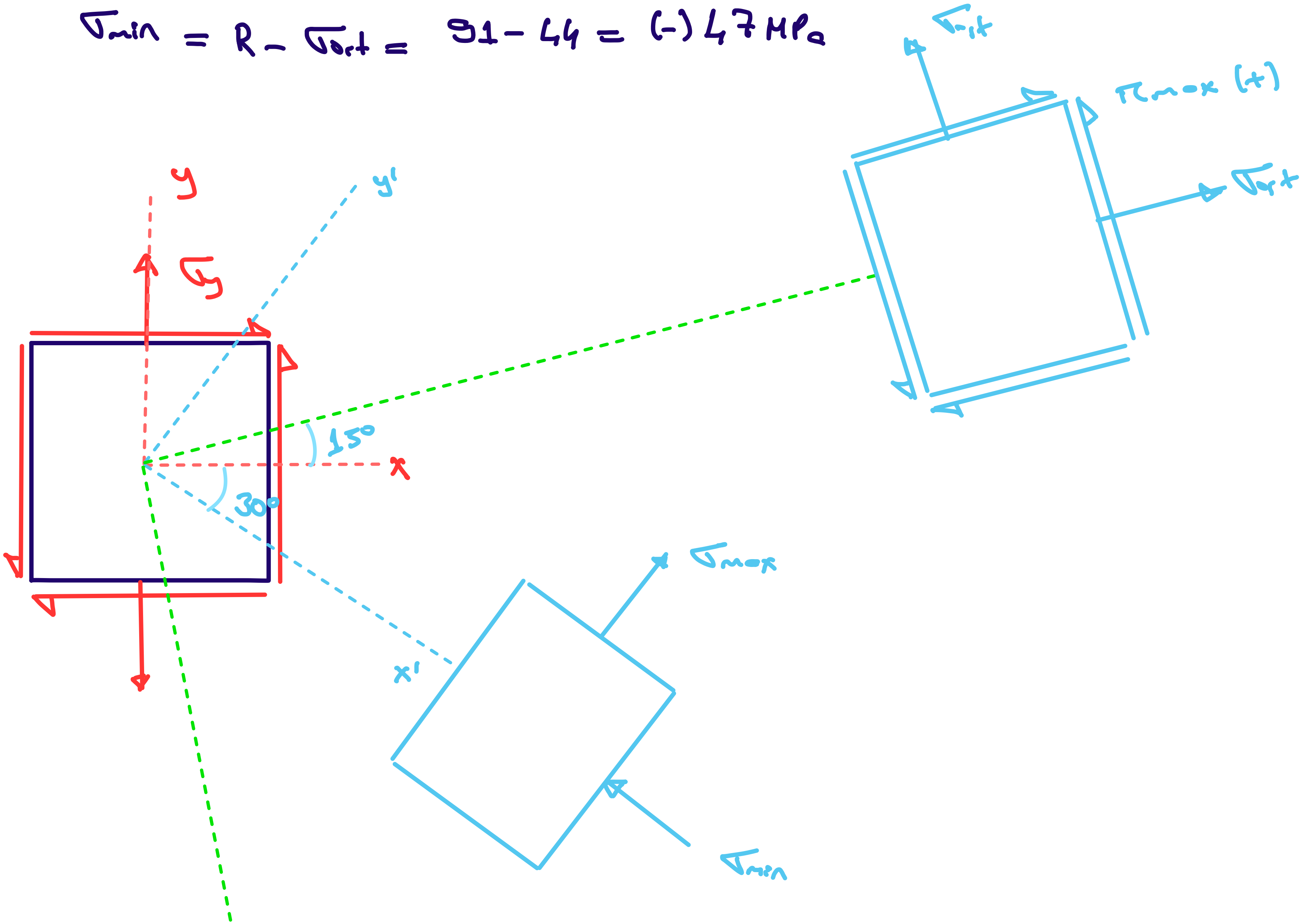
$$R^2 = 80^2 + 44^2 \Rightarrow R = 91 \text{ MPa}$$

$$= \tau_{max}$$

$$(\sigma = \sigma_{ort})$$

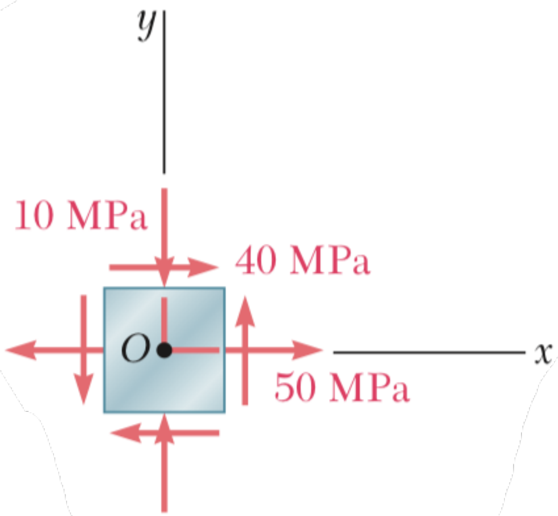
$$\sigma_{max} = R + \sigma_{ort} = 91 + 44 = 135 \text{ MPa}$$

$$\sigma_{min} = R - \sigma_{ort} = 91 - 44 = (-) 47 \text{ MPa}$$



EXAMPLE 7.02

For the state of plane stress already considered in Example 7.01, (*a*) construct Mohr's circle, (*b*) determine the principal stresses, (*c*) determine the maximum shearing stress and the corresponding normal stress.

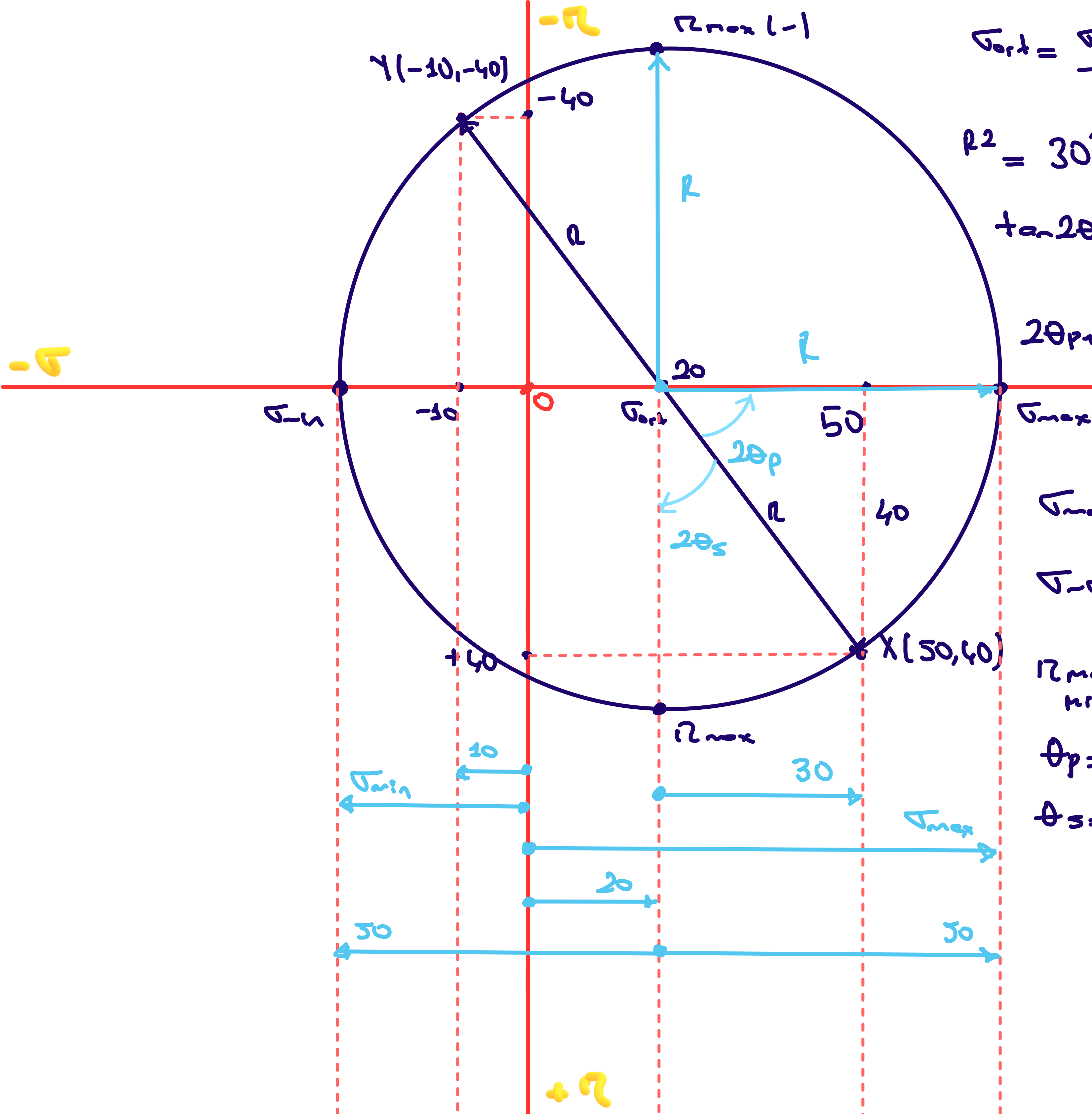


(*a*)

$\sigma_x = 50$ MPa $\quad X(\sigma_x, (+)\tau_{xy})$

$\sigma_y = -10$ MPa $\quad Y(\sigma_y, (-)\tau_{xy})$

$\tau_{xy} = +40$ MPa $\Rightarrow X(50, 40) \quad Y(-10, -40)$



$$\sigma_{ort} = \frac{\sigma_x + \sigma_y}{2} = \frac{50-10}{2} = 20\,\text{MPa}$$

$$R^2 = 30^2 + 40^2 \Rightarrow R = 50\,\text{MPa} = \tau_{max}$$

$$\tan 2\theta_p = \frac{40}{30} \Rightarrow 2\theta_p = 53^\circ$$

$$\theta_p = 26.5^\circ$$

$$2\theta_p + 2\theta_s = 90^\circ \Rightarrow 2\theta_s = 37^\circ$$

$$\theta_s = 18.5$$

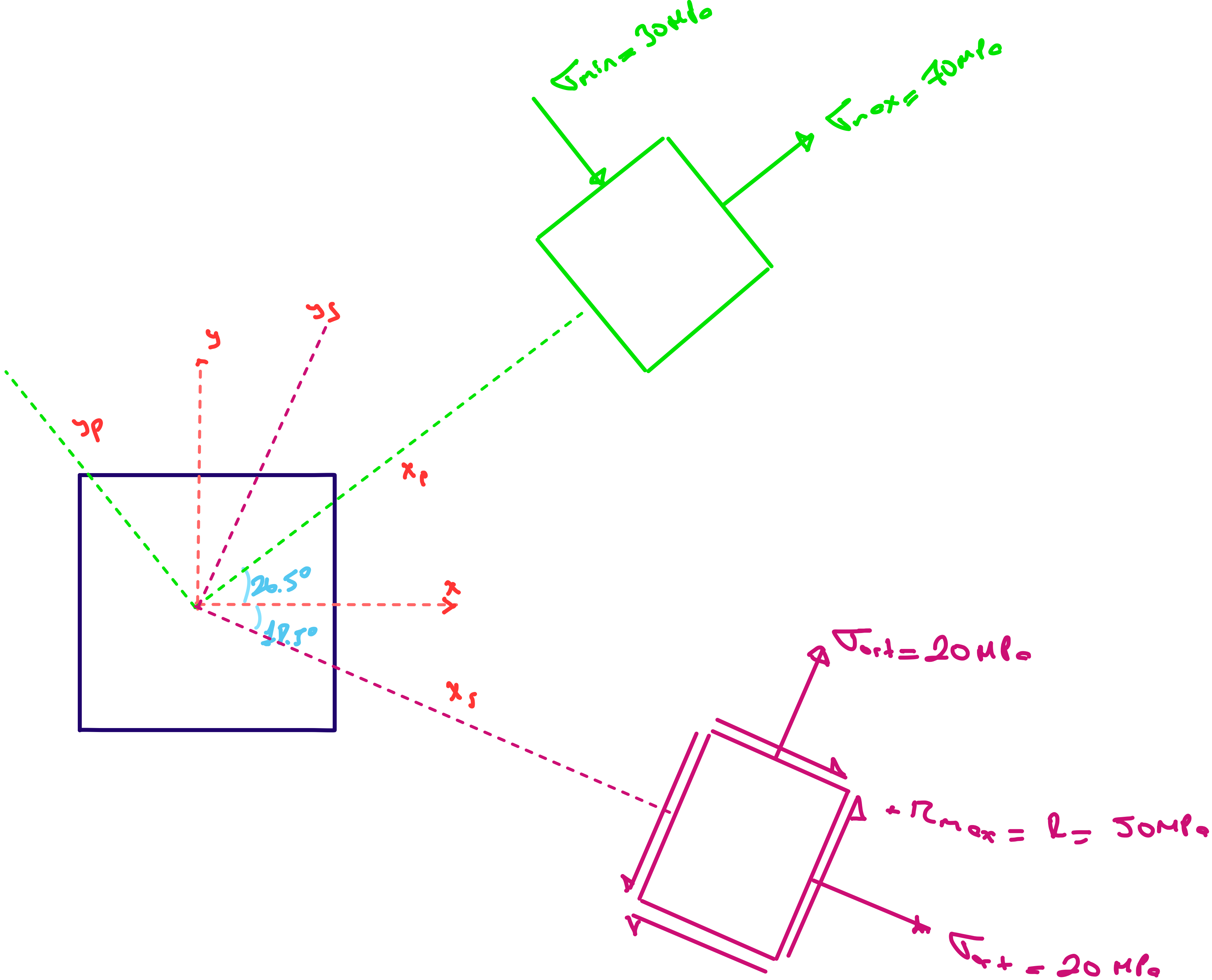
$$\sigma_{max} = 20 + 50 = 70\,\text{MPa}$$

$$\sigma_{min} = 50 - 20 = 30\,\text{MPa}\ (-)$$

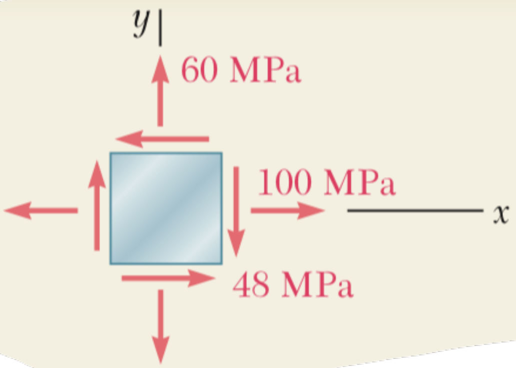
$$\tau_{\substack{max\\min}} = \pm 50\,\text{MPa}$$

$$\theta_p = 26.5^\circ$$

$$\theta_s = 18.5^\circ$$



σmin = 30MPa
σmax = 70MPa
yp
y
ys
xp
x
26.5°
18.5°
xs
σort = 20MPa
+τmax = R = 50MPa
σort = 20 MPa



SAMPLE PROBLEM 7.2

For the state of plane stress shown, determine (*a*) the principal planes and the principal stresses, (*b*) the stress components exerted on the element obtained by rotating the given element counterclockwise through 30°.

$\sigma_x = +100\,MPa$ $\tau_{xy} = -48\,MPa$ $X(100, -48)$ $Y(60, 48)$

$\sigma_y = +60\,MPa$

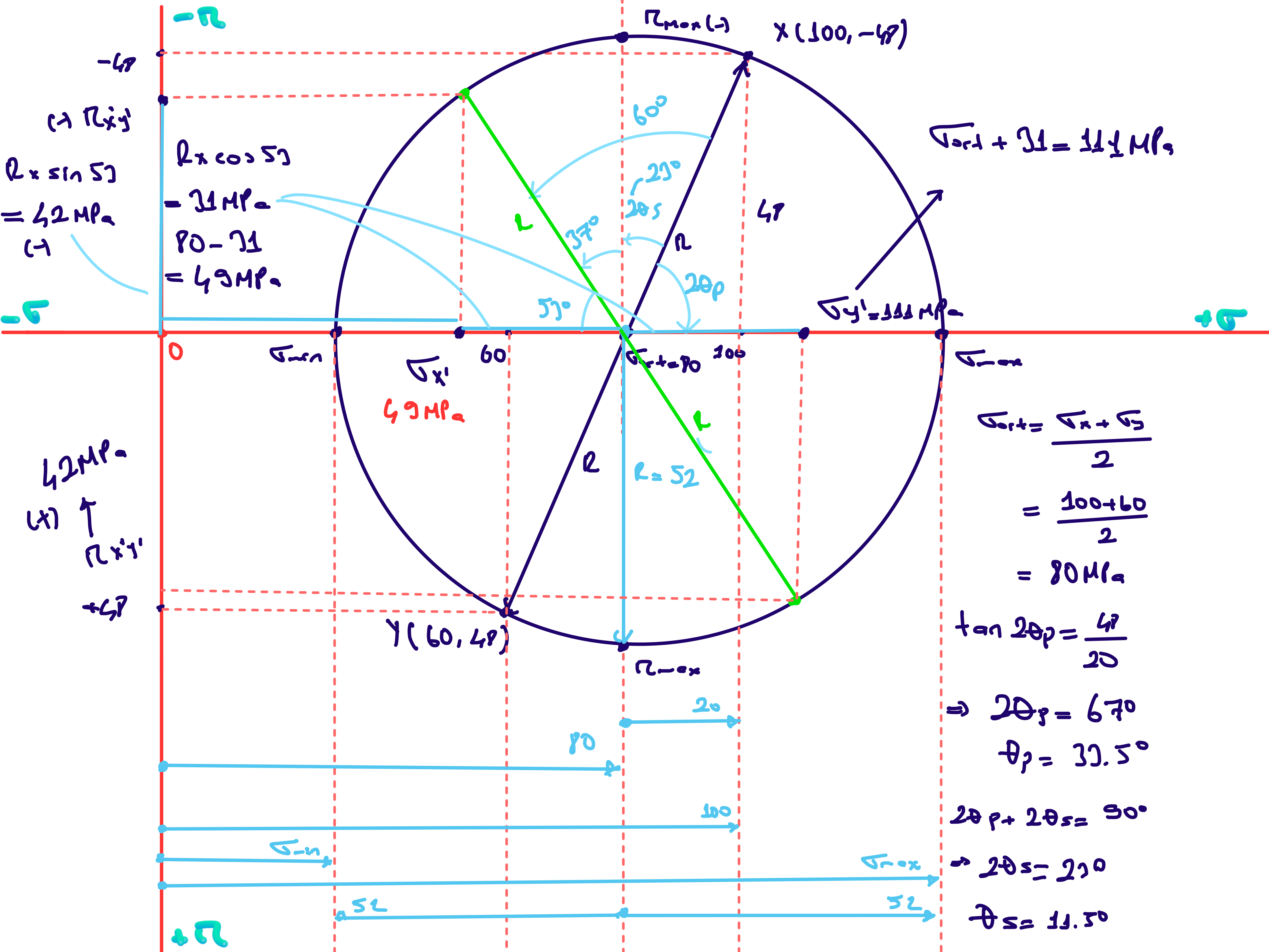
GERCEKTE

30°

MOHR

$2 \times 30^\circ$

$= 60^\circ$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{100+60}{2} = 80\ \text{MPa}$$

$$\tan 2\theta_p = \frac{48}{20}$$

$$\Rightarrow 2\theta_p = 67^\circ$$

$$\theta_p = 33.5^\circ$$

$$2\theta_p + 2\theta_s = 90^\circ$$

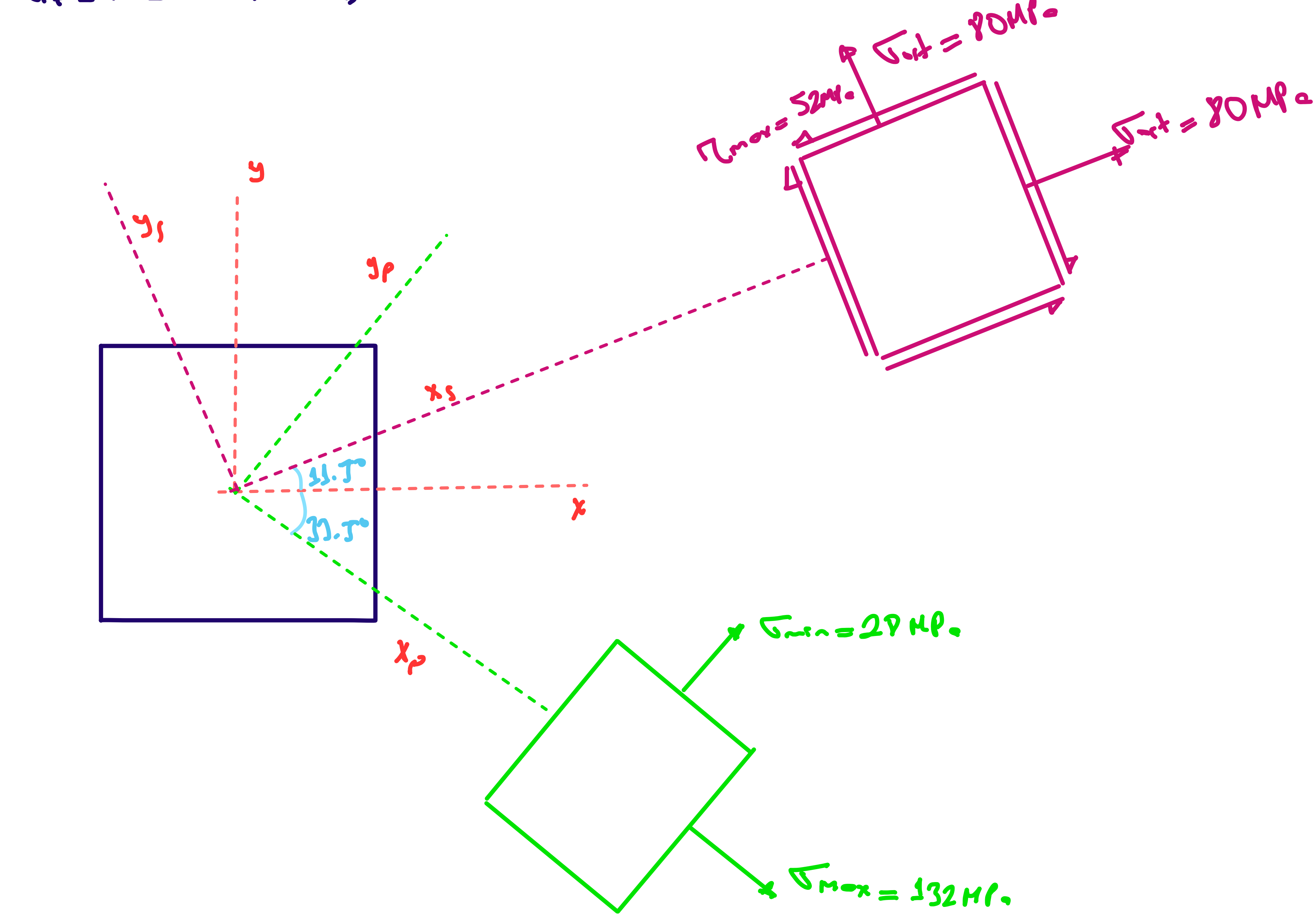
$$\Rightarrow 2\theta_s = 23^\circ$$

$$\theta_s = 11.5^\circ$$

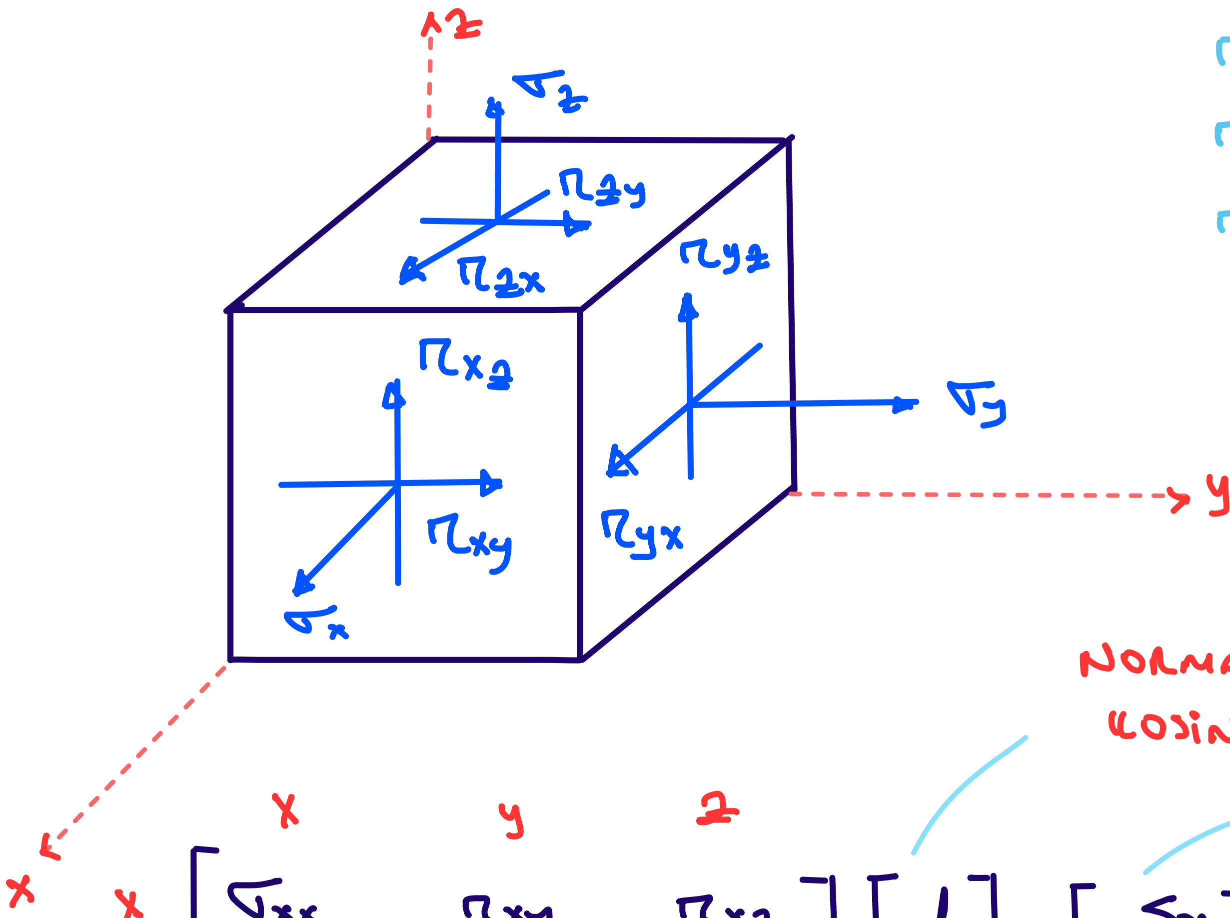
$R^2 = 20^2 + 48^2 \Rightarrow R = 52\,MPa = \tau_{max}$

$\sigma_{ort} + R = \sigma_{max} \Rightarrow 80 + 52 = 132\,MPa = \sigma_{max}\,(+)$

$\sigma_{ort} - R = \sigma_{min} \Rightarrow 80 - 52 = 28\,MPa = \sigma_{min}\,(+)$



3 BOYUTLU GERİLME HALİ



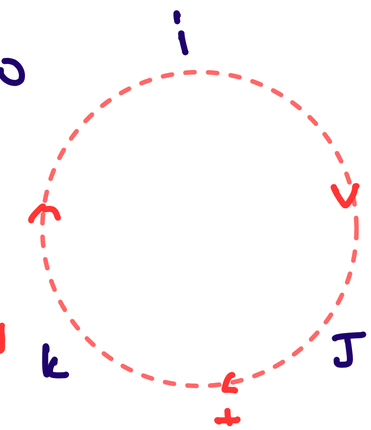
$\tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

TOPLAM 6 GERİLME VAR

$i^2 = k^2 = j^2 = 0$



NORMALİN DOĞRULTMAN KOSİNÜSLERİ

GERİLME BİLEŞENLERİ

$$
\begin{array}{c|ccc}
 & x & y & z \\
\hline
x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\
y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\
z & \tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{array}
\qquad
\underbrace{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}}_{\text{GERİLME TENSÖRÜ}}
\begin{bmatrix} l \\ m \\ n \end{bmatrix}
=
\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix}
$$

$l = \cos\theta_x$

$m = \cos\theta_y$

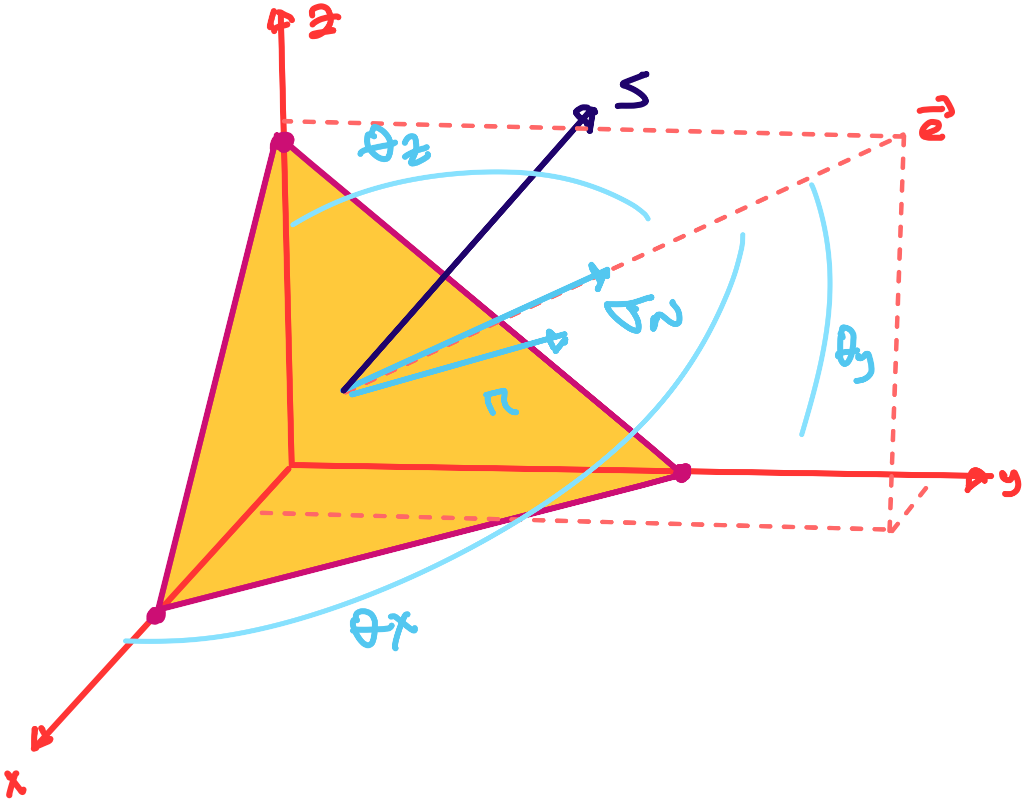
$n = \cos\theta_z$

$\vec{e} = l\vec{i} + m\vec{j} + n\vec{k}$

$l^2 + m^2 + n^2 = 1$

$\vec{S} = S_x\vec{i} + S_y\vec{j} + S_z\vec{k}$

$S = \sqrt{S_x^2 + S_y^2 + S_z^2}$

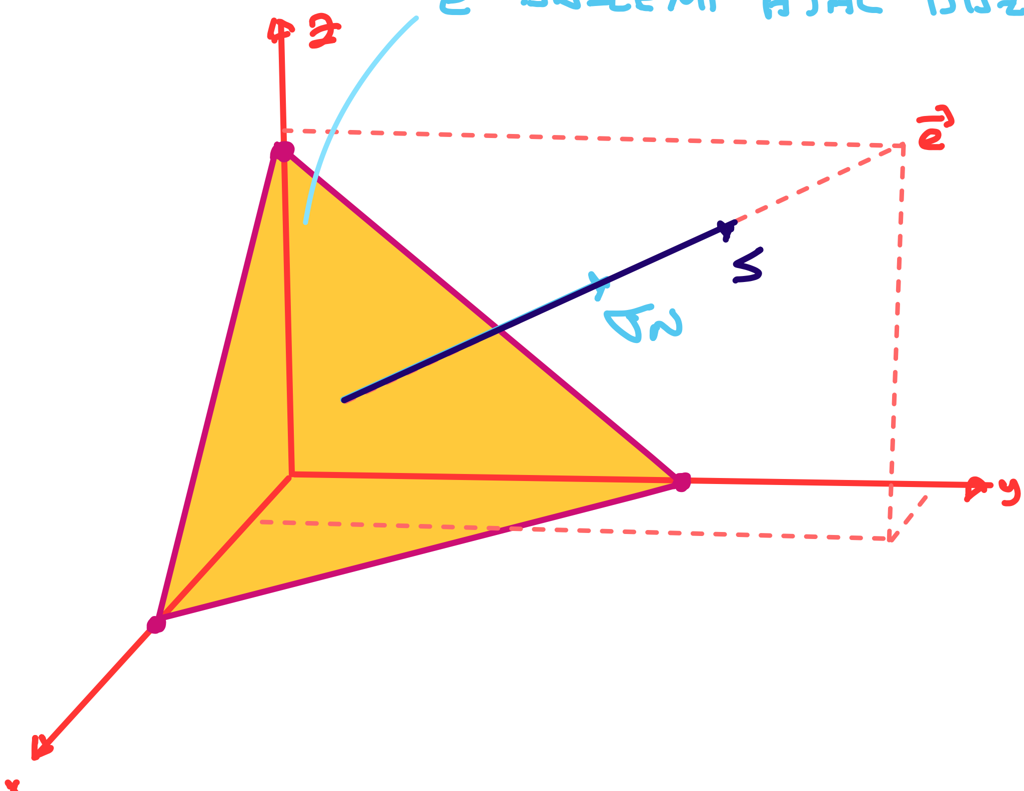


$$S = \sqrt{\sigma_N^2 + r^2}$$

$$\vec{S} \times \vec{e} = \sigma_N$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

e DÜZLEMİ ASAL DÜZLEM



$$[\sigma] \times \vec{e} = \vec{\sigma_p}$$

$$\sigma_N \times \vec{e} = \vec{\sigma_p}$$

SKALER

$$[\sigma] \times \vec{e} = \sigma_N \times \vec{e}$$

$$[\sigma] \times \vec{e} - \sigma_N \times \vec{e} = 0$$

$$([\sigma] - \sigma_N . I)\vec{e} = 0$$

ÖRNEK

σ_1 için

$$\underbrace{\begin{bmatrix} \sigma_{xx}-\sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma_p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}-\sigma_p \end{bmatrix}}_{\det = 0} \underbrace{\begin{bmatrix} l \\ m \\ n \end{bmatrix}}_{\text{SIFIR OLAMAZ}} = 0$$

σ_P

$$+\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

, $I_1 - I_2 - I_3$: Gerilme invaryantları

- $I_1 = \sigma_x + \sigma_y + \sigma_z$ (MPa)
- $I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2$ (MPa^2)
- $I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - (\sigma_x\tau_{yz}^2 + \sigma_y\tau_{xz}^2 + \sigma_z\tau_{xy}^2)$ (MPa^3)

DENKLEMİN KÖKLERİ $\longrightarrow$ $\sigma_1 > \sigma_2 > \sigma_3$

1. YOL

- $a_i = \text{Minör} + (\sigma_x - \sigma_i)$

$b_i = \text{Minör} - (\tau_{xy})$

$c_i = \text{Minör} + (\tau_{xz})$

↳ Determinant

$$k_i = \frac{1}{\sqrt{a_i^2 + b_i^2 + c_i^2}}$$

$l_i = a_i k_i \qquad m_i = b_i k_i \qquad n_i = c_i k_i$

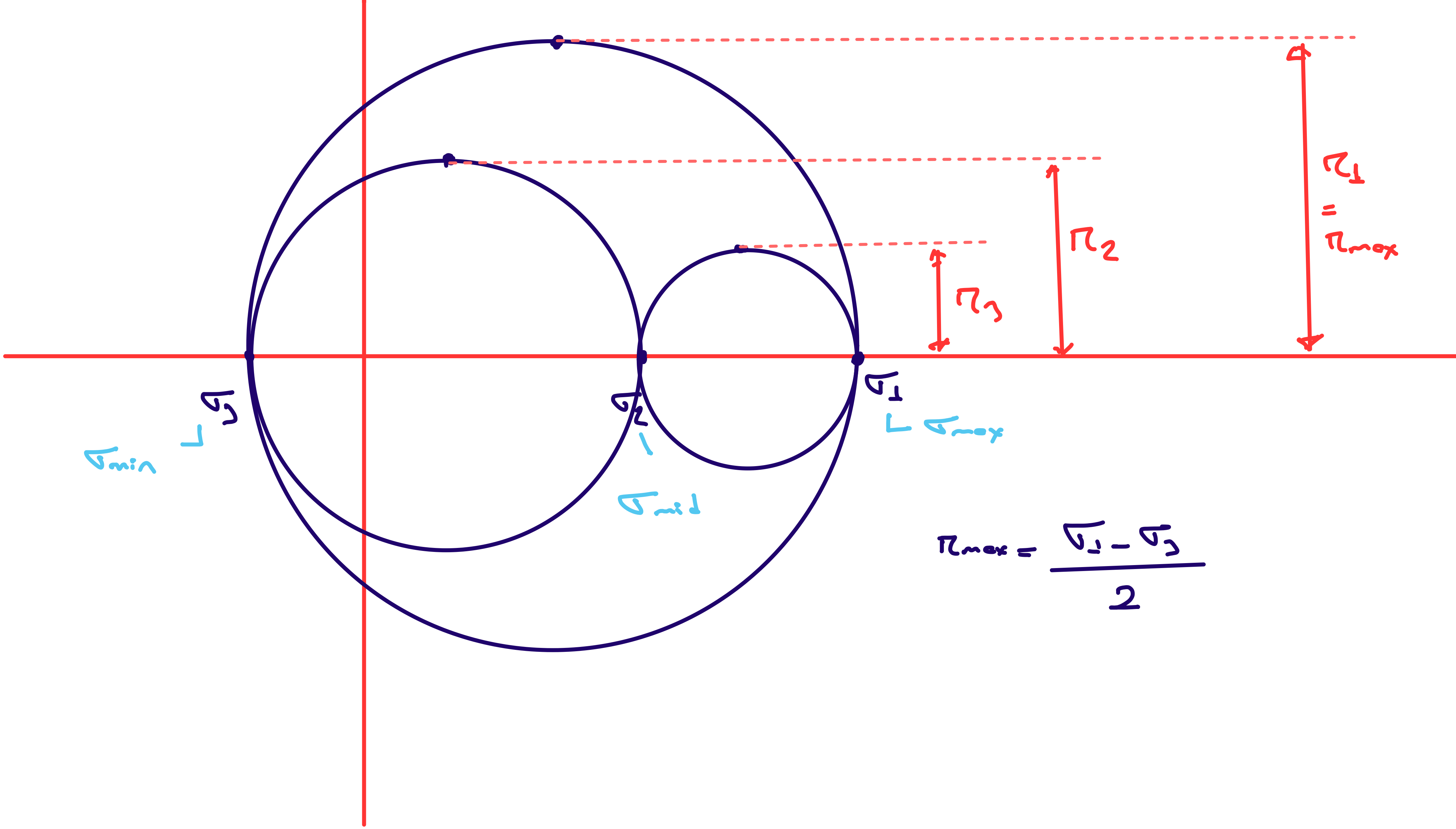
2. YOL

$$\left.\begin{aligned} (\sigma_x - \sigma_i)\, l_i + \tau_{xy}\, m_i + \tau_{xz}\, n_i &= 0 \\ \tau_{xy}\, l_i + (\sigma_y - \sigma_i)\, m_i + \tau_{xz}\, n_i &= 0 \\ \tau_{zx}\, l_i + \tau_{zy}\, m_i + (\sigma_z - \sigma_i)\, n_i &= 0 \end{aligned}\right\} (1)$$

$$l_i^2 + m_i^2 + n_i^2 = 0 \quad \} (2)$$

(1) – (2) → l_i, m_i ve n_i bulunur.

↳ l_i, m_i ve n_i 'deki i σ_i'deki i'dir. i, j ve k'daki i değil.



$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$